ARMY RESEARCH LABORATORY

Wind Drift of Projectiles: A Ballistics Tutorial

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ARL-TR-1124

October 1996

NOTICES

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
October 1996

3. REPORT TYPE AND DATES COVERED
Technical Report

4. TITLE AND SUBTITLE

WIND DRIFT OF PROJECTILES: A BALLISTICS TUTORIAL

5. FUNDING NUMBERS

6. AUTHOR(S)

Herbert A. Leupold

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
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Physical Sciences Directorate (PSD)
ATTN: AMSRL-PS-PC
Fort Monmouth, NJ 07703-5601

8. PERFORMING ORGANIZATION REPORT NUMBER

ARL-TR-1124

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for Public Release: Distribution is Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Projectile trajectories are altered in the presence of wind. This treatise discusses the lateral displacement of a projectile by a wind blowing at right angles to the direction of aim and explains the reasons for the same. The effect of bullet shape and muzzle velocity upon the trajectory and the resulting wind drift is included.

14. SUBJECT TERMS
wind drift, muzzle velocity, air resistance, projectile, sectional density, form factor

15. NUMBER OF PAGES
10

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-8
298-01

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WIND DRIFT OF PROJECTILES: A BALLISTICS TUTORIAL

Herbert A. Leupold

Derivation of Wind Drift Formula

The lateral displacement of x_d of a projectile by a wind blowing at right angles to the direction of aim is given by the well known formula:

$$x_d = V_w(t_a - t_v) \tag{1}$$

where V_w is the wind velocity, t_a is the projectile's time of flight from the launcher to the target and t_v its time of flight if fired in a vacuum.

At first glance, this formula is disturbing to many because of the suggestion of mysticism it seemingly presents, viz: How does the projectile "know" what its time of flight should be in a vacuum and why should it affect the projectile?

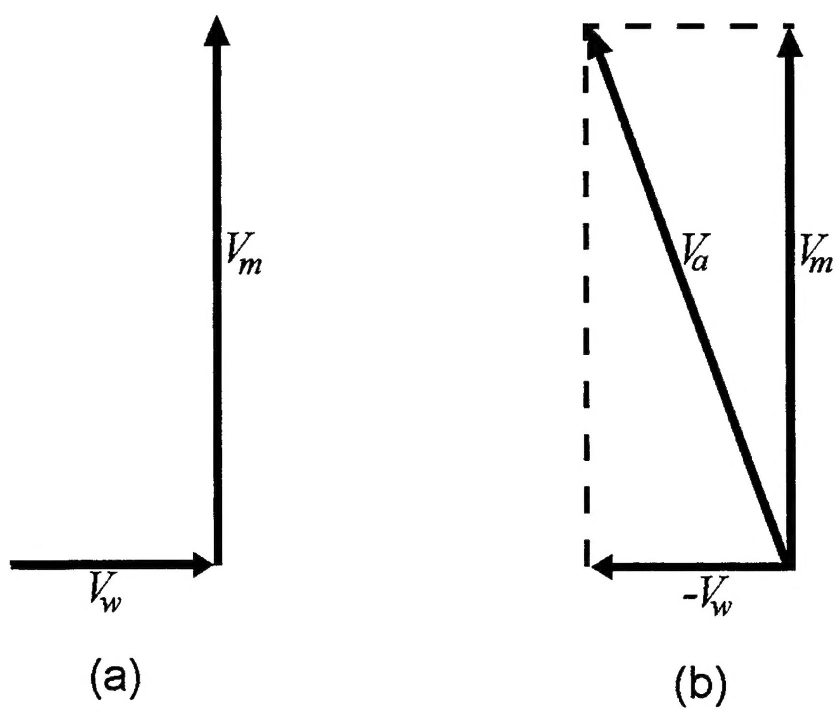

Figure 1: (a) Wind velocity, V_w, and projectile velocity, V_m, in ground reference frame, (b) Projectile velocity in air reference frame.

Fortunately the appearance of t_v in (1) is only an accident of geometry and does not imply any causal link between x_d and t_v.

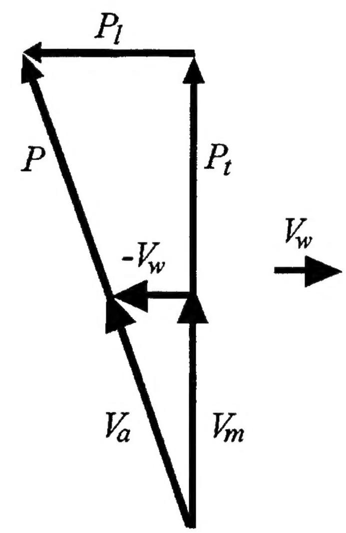

Figure 2: Projectile path and its components form triangles similar to those of the initial and all subsequent components of velocity.

The best way to illustrate this is to consider the motion of the projectile in the reference system in which the air is stationary. In that frame the force the air exerts upon the projectile is opposite to its direction of motion. Initially the components of projectile velocity V_a with respect to the air are the muzzle velocity, V_m and minus the wind velocity, $-V_w$. See Fig. 1.

The direction of flight in the air system does not change with time since the air exerts no force normal to the projectile's path in that reference frame. Therefore the projectile path and its components form triangles similar to those of the initial and all subsequent components of velocity as in Fig. 2. Here P is the path taken by the projectile in the air reference frame, P_l is the lateral displacement at the target of the projectile and P_t the distance of the launch site to the target. From Fig. 2 we form the proportion:

$$\frac{P_l}{P_t} = \frac{V_w}{V_m}$$
$$P_l = \frac{V_w}{V_m} P_t \tag{2}$$

In the time the projectile has traversed path P in the air system, the air has moved a distance $V_w t_a$ to the right. From this we must subtract the lateral displacement, P_l of the projectile with respect to air to obtain the lateral drift, x_d, with respect to the ground. Doing this we obtain:

$$x_d = V_w t_a - P_l = V_w t_a - \frac{V_w P_t}{V_m} = V_w \left(t_a - \frac{P_t}{V_m} \right) \tag{3}$$

But P_t / V_m is just the range P_t divided by the muzzle velocity V_m, which is the time of flight t_v in a vacuum since in a vacuum projectile velocity remains constant with the value V_m. Finally we obtain (1):

$$x_d = V_w (t_a - t_v) \tag{4}$$

Thus we see that the appearance of t_v in (4) is a geometrical artifact that arises because the range P_t can be written as $V_m t_v$.

Effect of Projectile Shape

If the projectile is a sphere it will always present the same cross section normal to its path in the air frame of reference, i.e., a circular one (Fig. 3).

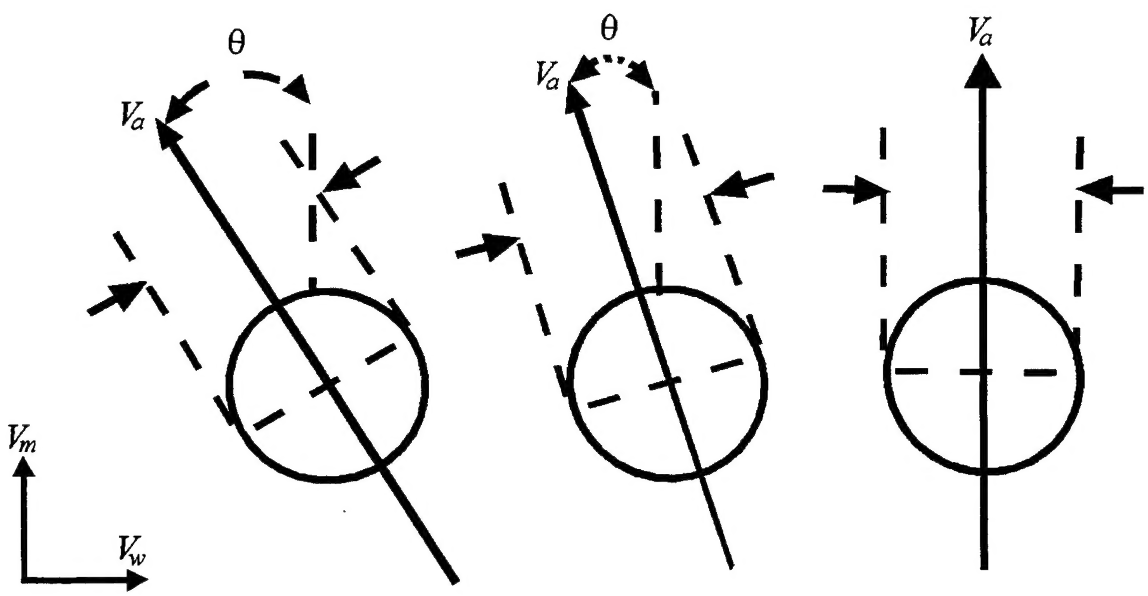

Figure 3: The angles, θ, have been exaggerated for clarity. In actual cases V_m is always much larger than V_w and hence θ is small. (A) high wind velocity, (B) medium wind velocity, (C) zero wind velocity.

If, however, the projectile is of the usual pointed cylinder form it would seem that the cross section impacted by the air's relative motion varies with wind velocity as in Fig. 4.

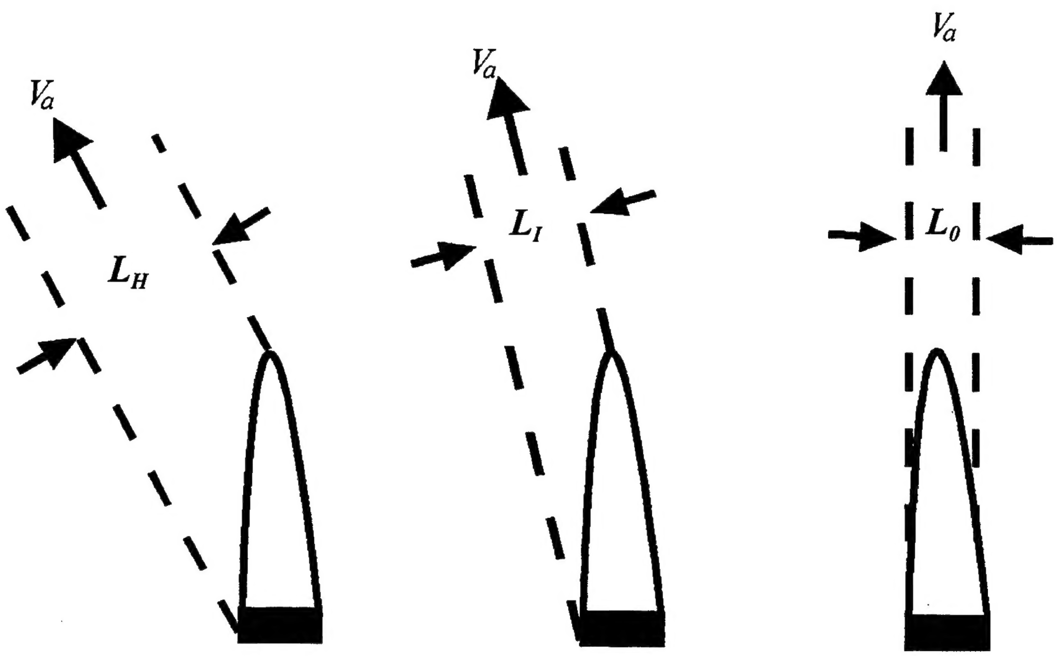

Figure 4: (A) high wind velocity, (B) intermediate wind velocity, (C) zero wind velocity.

This means that the time of flight would be affected by wind velocity since the three relative airspeed orientations, Fig. 4(A), (B), (C), result in different air resistance and, hence, in different times of flight. The cases (A) to (C) result in decreasing times of flight because the cross-sectional areas, L, presented to the oncoming air decrease in that sequence.

This complication does not occur in practice because a spinning projectile such as that from a rifle will always precess about the direction of air-resistance, i.e., in the direction opposite to the projectile's air speed thus automatically orienting itself point onward. The vanes of non-spinning dart-like projectiles produce the same effect.

Effect of Muzzle Velocity on Wind Drift

If the muzzle velocity is increased, both t_a and t_v decrease. At most muzzle velocities and ranges actually used this results in a decrease in $(t_a - t_v)$ itself, indicating less wind drift. But at low velocities and for some ballistic shapes it is actually possible for the decrease in t_v to exceed the drop in t_a with increased muzzle velocity so that the wind drift becomes greater. This is a problem for projectiles from 22 calibre rim fire target rifles wherein target ammunition is actually loaded to velocities below the standard to minimize wind effects.

Effect of Bullet Shape and Mass on Wind Drift

A sharply pointed bullet that is aerodynamically shaped loses velocity much more slowly than a flat or round nosed bullet because it encounters less air resistance and so has a higher average velocity over a given range. This means that at equal muzzle velocity t_a will be shorter than for a less well shaped bullet, and since t_v is the same for both projectiles, Δt, and hence the wind drift, x_d, will also be less.

The deceleration of a of the bullet due to air resistance f is given by Newton's law:

$$a = f / m \tag{5}$$

where m is the bullet mass. The force f is proportional to A, the cross-sectional area of the bullet presented to the air. So from (5) we have:

$$a = k \; A / m \tag{6}$$

where k is a constant of proportionality equal to an average pressure on the projectile. The quantity m/A is called the sectional density and since the bullet velocity loss is inversely proportional to it, it should be as large as possible to minimize wind drift. Finally, equation (6) takes on a particularly simple form if expressed in terms of the average density of the bullet, σ. Then bullet mass is bullet volume, V, multiplied by σ or:

$$m = \sigma V = \sigma \, Al \tag{7}$$

where l is the average length of the bullet and the sectional density, s, becomes:

$$s = m / A = \sigma \, Al / a = \sigma \, l \tag{8}$$

So the sectional density is the average length of a bullet times its density, and if all bullets are made of the same material, lead, a bullet's efficiency in cutting through air can be measured by its mean length.

In summary, we note that to minimize wind drift one should:

(1) Use a bullet of the best possible aerodynamic shape or form factor.

(2) Maximize sectional density consistent with bullet stability and attainable velocity.

(3) Use the highest possible muzzle velocity for a bullet best satisfying (1) and (2) at the longer ranges with high velocity centerfire rifles.

This is because the time of flight t_a is a complex function of these bullet quantities and of the initial velocity. In practice t_a is usually obtained from ballistic tables where it is tabulated for various form factors, sectional densities and muzzle velocities.

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